

Universal scaling in quenches across a discontinuity critical point

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We study slow variation (both spatial as well as temporal) of a parameter of a system in the vicinity of discontinuous quantum phase transitions, in particular, a discontinuity critical point (DCP) (or a first-order critical point). We obtain the universal scaling relations of the density of defects and the residual energy after a temporal quench, while we also unravel the scaling of the characteristic length scale associated with a spatial quench of a symmetry breaking field. Considering a spin-1/2 XXZ chain we establish how these scaling relations get modified when the DCP is located at the boundary of a gapless critical phase; these predictions are also confirmed numerically.

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When a macroscopic system is driven from a disordered phase to an ordered phase by lowering the temperature across a phase transition, there is spontaneous generation of topological defects, namely, vortices, kinks and domain walls; this has been explained invoking upon the so called Kibble-Zurek (KZ) mechanism. Originally proposed in the context of the cosmic evolution [1, 2] and following the subsequent generalization to the condensed matter systems [3], in recent years the KZ mechanism has been extended to the quenches in the vicinity of a quantum phase transition [4–7]. In fact, the remarkable progress in experiments with cold atoms in optical lattices, which can be treated as nearly isolated systems, has paved the way for this recent upsurge in theoretical studies of non-equilibrium dynamics of closed quantum systems.

A number of studies have established that, when a d -dimensional quantum system is driven across an isolated quantum critical point (QCP), by changing a parameter of the Hamiltonian in a linear fashion as t/t_Q , the density of defects ($\hat{\rho}$) satisfies the KZ scaling $t_Q^{-d\nu/(z\nu+1)}$. Similarly when quenched to the QCP, the residual energy density, i. e., the excess energy density over the ground state of the final Hamiltonian (ε_{res}) scales as $t_Q^{-(d+z)\nu/(z\nu+1)}$; here, ν is the critical exponent of the correlation length, and z is the dynamical critical exponent associated with the corresponding QCP. On the contrary, when quenched to the gapped phase, the residual energy follows a scaling relation identical to that of the defect. These scaling formulae however get modified, sometimes drastically, in several interesting situations [8–13]. (see for review [14–16]).

The concept of the KZ scaling has also been extended to spatial quenches [17, 18], where a parameter is inhomogeneous along a spatial coordinate x characterised by a slope x_Q^{-1} with the quantum (or classical) phase transition occurring at $x = 0$. The order parameter is found to bend smoothly over the region $|x| \lesssim \hat{\xi}_s \sim x_Q^{\nu/(1+\nu)}$; obviously, $\hat{\xi}_s$ sets a length scale characterized by quantum critical exponents. These predictions has been confirmed

analytically as well as numerically in several models [17–19].

The present Letter studies temporal as well as spatial quenches near a quantum *discontinuous (first-order) phase transition*, which, to the best of our knowledge, has not received much attention in the context of quenching and the consequent KZ mechanism scenario; however, the time evolution across a discontinuous phase transition has been widely studied from the viewpoint of quantum annealing (or adiabatic quantum computation) mainly concentrating on the scaling of the energy gap [20, 21]. Although quite recently, Bonati *et al.* [22] proposed a scaling of the length scale $\hat{\xi}$ near a classical discontinuous transition in the presence of a temperature gradient, the same for a *sloped* symmetry breaking field is not known.

The goal of the present Letter is to show that $\hat{\rho}$ and ε_{res} after a temporal quench across (or at) a *discontinuity critical point* (DCP) (or a *first-order critical point* [23, 24]) do indeed follow universal scaling laws, given in terms of the associated critical exponents derived here for a generic DCP. On the contrary, when a symmetry breaking field is spatially quenched near the DCP, a characteristic length scale is also shown to emerge. Even though similar scaling relations can be derived for classical quenches, our focus will be restricted to quantum quenches only.

We initiate our discussion with a phenomenological description of discontinuous transitions, illustrating with the example of simple Ising-like systems with Z_2 symmetry exposed to a tunable quantum fluctuation with its strength denoted by Γ . Due to Ising symmetry, doubly degenerate ordered ground states coexist at a low Γ in the absence of a symmetry breaking field. With increasing Γ , the order parameter characterizing the ordered states vanishes discontinuously at a discontinuous transition Γ_c . Application of a symmetry breaking field h , on the other hand, lifts the degeneracy of two states with nonzero order parameter; as h is tuned across the zero-field coexisting line (see Fig. 1), the order parameter shows a jump from one to the other. One may now

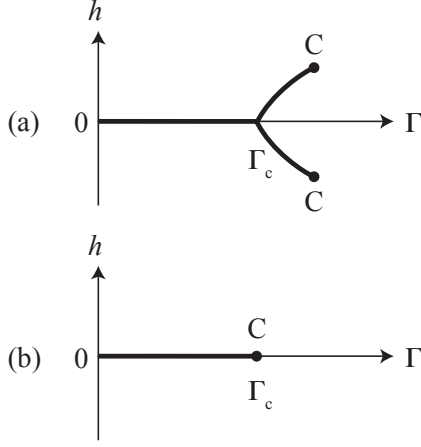


FIG. 1: Schematics of two phase diagrams involving a discontinuous transition.

wonder what is the fate of this coexisting line beyond Γ_c ? There are indeed two possibilities: in the former, the coexisting line splits into two lines (Fig. 1(a)) when the order parameter shows a discontinuous jump to a finite value even at $\Gamma > \Gamma_c$ with increasing the magnitude of h due to an interchange of a stable and a metastable states. The coexisting line ends at a critical point denoted by C in Fig. 1(a), beyond which metastable states disappear. This scenario can be explained within the framework of the phenomenological Ginzburg-Landau-Wilson theory. In the latter case, on the contrary, the coexisting line terminates at Γ_c (Fig. 1(b)) with no metastable state appearing for $\Gamma > \Gamma_c$. The transition point Γ_c is referred to as a DCP and is the focus point of the present Letter.

To enunciate the critical properties associated with a DCP, one proceeds with the following assumptions: at the DCP, the ground-state energy density $\varepsilon(\Gamma, h)$ is a continuous function of Γ and h , but its derivatives $\partial\varepsilon(\Gamma, 0)/\partial\Gamma$ and the order parameter $m(\Gamma, 0)$ (at $h = 0$) exhibit jumps as a function of Γ . We put them in a precise mathematical form:

$$\varepsilon(\Gamma_c + 0, 0) = \varepsilon(\Gamma_c - 0, 0) \quad (1)$$

$$\frac{\partial\varepsilon(\Gamma_c + 0, 0)}{\partial\Gamma} \neq \frac{\partial\varepsilon(\Gamma_c - 0, 0)}{\partial\Gamma}, \quad (2)$$

$$|m(\Gamma_c - 0, 0)| > m(\Gamma_c + 0, 0) = 0. \quad (3)$$

On the other hand, right at $\Gamma = \Gamma_c$, m shows a discontinuous change with h at $h = 0$:

$$m(\Gamma_c, +0) \neq m(\Gamma_c, -0). \quad (4)$$

Equipped with the above assumptions, let us now propose the scaling ansatz, first fixing $h = 0$: As usual, the correlation length (ξ) and the ground-state energy density near a quantum critical point follow scaling forms,

$\xi \approx A^{(\pm)}|\Gamma - \Gamma_c|^{-\nu}$ and $\varepsilon(\Gamma, 0) - \varepsilon(\Gamma_c, 0) \approx B^{(\pm)}\xi^{-(d+z)}$ [25, 26], where ν and z are the correlation length and the dynamical exponents respectively and \pm stands for the sign of $\Gamma - \Gamma_c$. The first order derivative of $\varepsilon(\Gamma, 0)$ is written as $\partial\varepsilon(\Gamma, 0)/\partial\Gamma \approx B^{(\pm)}\xi^{-(d+z)+1/\nu}$. This along with Eqs. (1) and (2) leads to $\nu = 1/(d+z)$. In the other situation, when $\Gamma = \Gamma_c$ and $h \neq 0$, the corresponding correlation length (ξ_H) and the ground-state energy density satisfy $\xi_H \approx A_H^{(\pm)}|h|^{-\nu_H}$ and $\varepsilon(\Gamma_c, h) - \varepsilon(\Gamma_c, 0) \approx B_H^{(\pm)}\xi_H^{-(d+z)}$. Recalling that m is determined by $m(\Gamma, h) = -\partial\varepsilon(\Gamma, h)/\partial h$, these relations together with Eq. (4) yields $\nu_H = 1/(d+z)$.

Henceforth, we move on to the non-equilibrium situation arising due to quantum quenches. We consider a temporal quench of the parameter field (with $h = 0$) of the form $\Gamma(t) = \Gamma_c - \Gamma_0 \frac{t}{t_Q}$; here t is the time, t_Q^{-1} is a quench rate, and Γ_0 is a positive constant. Consequently, the system is driven from the disordered phase (for $t < 0$) to the ordered phase (for $t > 0$). Assuming a power-law scaling of the healing time $\tau(T, 0) \sim \xi(T, 0)^z$, for a time-varying parameter, we introduce the notion of the *instantaneous healing time* $\tau(t) = \xi(\Gamma(t), 0)^z$; using $\xi(\Gamma, 0) \sim |\Gamma - \Gamma_c|^{-\nu}$ and the quenching form of Γ , this can be written in the form $\tau(t) \sim |t/t_Q|^{-z\nu}$. If the system is initially prepared in the ground state, it remains in the instantaneous ground state as long as the healing time is short i.e., the system is far away from the QCP at $t = 0$. This adiabatic time evolution breaks down, when $t \rightarrow 0$ as a consequence of the diverging healing time. In order to derive the scaling of the healing length, we invoke upon the adiabatic-impulse approximation [7] which assumes that the system is in the instantaneous ground state when $t < -\tau(t)$, while it freezes when $t > -\tau(t)$. The freezing time \hat{t} is then determined by the condition $|\hat{t}| = \tau(\hat{t})$. One then immediately finds $\hat{t} \sim t_Q^{z\nu/(z\nu+1)} = t_Q^{z/(d+2z)}$. For $t > \hat{t}$, the system has an inhomogeneity which is characterized by a length scale $\hat{\xi}$, which can be derived from the scaling relation

$$\hat{\xi}_t \approx \xi(\Gamma(\hat{t}), 0) \sim t_Q^{1/(d+2z)}. \quad (5)$$

Noting that the defects are distributed in a d dimensional phase space, the scaling of the density of defects is then given by $\hat{\rho} \sim \hat{\xi}_t^{-d}$, which yields

$$\hat{\rho} \sim t_Q^{-d/(d+2z)}. \quad (6)$$

If the elementary excitation in the system after a temporal quench has a gapless dispersion as $|\mathbf{k}|^z$ with the wave vector \mathbf{k} , the residual-energy density follows the scaling relation

$$\varepsilon_{\text{res}} \sim t_Q^{-(d+z)/(d+2z)}; \quad (7)$$

otherwise, it satisfies the scaling given in Eq. (6).

In a similar spirit, let us now consider a spatial quench of a parameter of the Hamiltonian, $h(x) = h_0(x/x_Q)$,

keeping Γ fixed at Γ_c , where x refers to one of the d spatial coordinates, x_Q^{-1} is the quench rate or the scale of inhomogeneity, and h_0 is a positive constant. Note that the system is critical when the field vanishes, *i.e.*, at around $x = 0$. Defining the local correlation length [18] $\xi(x) = \xi(\Gamma_c, h(x))$ and using the relation $\xi(\Gamma_c, h) \sim |h|^{-\nu_H}$ ($\nu_H = 1/(d+z)$), one readily finds $\xi(x) \sim (|x|/x_Q)^{-\nu_H}$. In spite of the inhomogeneity, the system should maintain the local equilibrium for x sufficiently far from $x = 0$ since the local correlation length becomes much smaller than x_Q . On approaching the critical situation, however, the local correlation grows and hence the approximation of the local equilibrium breaks down leading to a finite correlation length near $x = 0$. To make an estimate of this finite correlation length, one again invokes upon the pulse-impulse approximation: the system maintains its local equilibrium for x satisfying $|x| \gg \xi(x)$, while for $|x| \ll \xi(x)$ the system *freezes* with the correlation length $\hat{\xi}$. The boundary \hat{x} of these two regimes is approximately determined by the condition $\xi(\hat{x}) = |\hat{x}|$ and $\hat{\xi}$ is given by $\hat{\xi}_s = \xi(\hat{x})$ which results in the scaling relation $|\hat{x}| \sim x_Q^{\nu_H/(1+\nu_H)} = x_Q^{1/(d+z+1)}$. One therefore arrives at the scaling relation of the length scale

$$\hat{\xi}_s \sim x_Q^{1/(d+z+1)}. \quad (8)$$

Equations (5), (7) and (8) constitute the main result of our paper regarding the quenching in the vicinity of a generic DCP as depicted in Fig 1(b). We generalize these relations below by exploring the dynamics of a XXZ chain in a magnetic field described by the Hamiltonian

$$H = \sum_{i=1}^{L-1} (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z) - \sum_{i=1}^L h_i \sigma_i^z, \quad (9)$$

where σ_i^α 's ($\alpha = x, y, z$) are the Pauli operators and L is the system size. We first set $h_i = 0, \forall i$, when the model exhibits a discontinuous quantum phase transition at $\Delta = -1$. The ground state for $\Delta < -1$ possesses the complete ferromagnetic order, while it is a disordered state for $-1 < \Delta < 1$. Starting from the ground state of the initial Hamiltonian with $\Delta = 0$, we consider a slow ramp to $\Delta = -1$ following a linear protocol $\Delta(t) = -t/t_Q$. Since the Hamiltonian commutes with $\sigma_{\text{tot}}^z = \sum_{i=1}^L \sigma_i^z$ and the initial state lies in the $\sigma_{\text{tot}}^z = 0$ sector, the dynamics is restricted within this manifold only. Even then the transition occurring at $\Delta = \Delta_c = -1$ *indeed* can be viewed as a DCP as we have established rigorously in the supplemental section. However, this DCP occurs at the end of a gapless critical line, which results in modification of associated critical exponents and consequently, of the scaling relations of $\hat{\rho}$ and ε_{res} .

The low-energy states of the XXZ chain with $-1 < \Delta \leq 1$ are described by the Tomonaga-Luttinger liquid with gapless excitations [27, 28]; this gaplessness, rules out the possibility of defining the exponent ν using the

conventional scaling of the energy gap $\Delta E \sim (\Delta - \Delta_c)^{z\nu}$ [25]. However, the dispersion relation at Δ_c is quadratic, *i.e.*, energy gap $\Delta E \sim L^{-2}$ implying $z = 2$.

To estimate the exponent ν , let us recall that the ground-state energy density in the $\sigma_{\text{tot}}^z = 0$ manifold behaves as $\varepsilon(\Delta) - \varepsilon(\Delta_c) = -(\Delta_c - \Delta)$ for $\Delta < -1$ and $\approx (\Delta - \Delta_c)^{3/2}$ for $\Delta > -1$ [29]. Comparing with the conventional scaling of the same, namely, $(\Delta - \Delta_c)^{\nu(d+z)}$ along with $d = 1, z = 2$, one finds that $\nu = 1/2$ which clearly differs from the predicted value $\nu = 1/(d+z)$. This mismatch stems from the existence of the gapless line for $\Delta > \Delta_c$ characterized by gapless excitations with dispersion $|k|^{z_+}$, with $z_+ = 1$ in the present case.

To settle this anomaly, we propose the following modified scaling forms of the spectrum $\Delta E \sim |\Delta - \Delta_c|^{\theta_\pm} k^{z_\pm} + k^z$ and the ground-state energy density $\varepsilon(\Delta) - \varepsilon(\Delta_c) \sim |\Delta - \Delta_c|^{\nu z} (1 + A_\pm |\Delta - \Delta_c|^{-\nu})^{-d}$, where z_\pm characterize the excitations on either sides of the DCP respectively; $z_+ = 1$ and $z_- = 0$ (on the gapped side), in the present case. Similarly, A_\pm are the critical amplitudes (of ξ) on the gapless and the gapped sides. The case $A_- = 0$ implies a perfectly ordered ground state without any fluctuation; this is indeed true for the XXZ chain which can be established using the same scaling of the ground-state energy density and the gap.

In order to derive the critical exponents of the DCP associated with a gapless phase, it is convenient to introduce a quantity $\phi(\Delta) = (\varepsilon(\Delta) - \varepsilon(\Delta_c))|\Delta - \Delta_c|^{-\nu z_\pm}$ with z_- for $\Delta < \Delta_c$ and z_+ for $\Delta > \Delta_c$, and enforce the modified conditions for the DCP in terms of $\phi(\Delta)$: (i) $\phi(\Delta)$ follows the same scaling law for $\Delta < \Delta_c$ and $\Delta > \Delta_c$, (ii) $\phi(\Delta)$ is continuous at $\Delta = \Delta_c$, and (iii) $\frac{\partial}{\partial \Delta} \phi(\Delta)$ is discontinuous at $\Delta = \Delta_c$ (see Supplemental Material for details). With these conditions together with the assumption that the gap closes on approaching the DCP as $|\Delta - \Delta_c|^{\nu z}$, one immediately finds: when $0 = A_- < A_+$, $z_- = d - z_+$, $\theta_\pm = \nu(z - z_\pm)$, and $\nu = 1/(d+z-z_+) = 1/(z-z_-)$, while if $A_\pm > 0$, $z_- = z_+$, $\theta_\pm = \nu(z - z_\pm)$, and $\nu = 1/(d+z-z_+)$. Remarkably, this reduces to $\nu = 1/2$ for the DCP in XXZ model where $A_- = 0$, $A_+ > 0$ and $z_+ = 1$, and $\nu = 1/(d+z)$ as predicted earlier when $A_\pm > 0$ and $z_\pm = 0$.

The modified scaling relations ξ_t , $\hat{\rho}$ and ε_{res} , can then be readily arrived at. We find

$$\xi_t \sim t_Q^{1/(d+2z-z_+)}, \quad (10)$$

$$\hat{\rho} \sim t_Q^{-d/(d+2z-z_+)}, \quad (11)$$

and

$$\varepsilon_{\text{res}} \sim t_Q^{-(d+z)/(d+2z-z_+)}, \quad (12)$$

respectively. For the DCP at Δ_c , $\theta_+ = 1/2$, $z = 2$ and $z_+ = d = 1$, one gets $\varepsilon_{\text{res}} \sim t_Q^{-3/4}$. We recover Eqs. (5) and (7) when $z_+ = 0$ which also implies $\theta_\pm = \nu z$.

This scaling relation is verified by numerically studying the time evolution using the time-dependent density

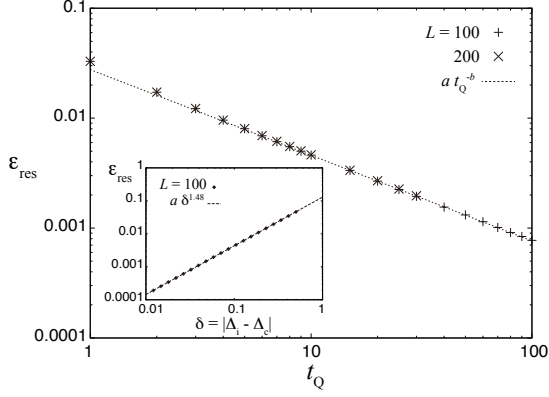


FIG. 2: Residual energy per spin of the XXZ chain after a temporal quench $\Delta(t) = -t/t_Q$ with time t from 0 to 1 and rate $1/t_Q$ in the absence of a magnetic field. Results of DMRG for sizes $L = 100$ and 200 and a fitting line are shown. Fitting is done by assuming a function $a t_Q^{-b}$ with the data of $L = 100$ and t_Q 's from 10 to 100 and yields $a = 0.0276$ and $b = 0.779$. Inset shows the scaling of ε_{res} for a sudden quench of small magnitude δ starting from the gapless phase and ending at the DCP.

matrix renormalization group (DMRG) for finite systems with the open boundary condition. We kept $m = 50$ states for each left and right blocks in DMRG and chose the Trotter slice $\Delta t = 0.025$ [32] with the initial state prepared by the static DMRG. Figure 2 shows the results for sizes $L = 100$ and 200 , confirming that ε_{res} indeed exhibits a power-law scaling with an exponent -0.779 ; this is consistent with the value $-3/4$ [33].

If the system is quenched across the QCP to the gapped ferromagnetic phase, the scaling of ε_{res} (which is identical to that of $\hat{\rho}$ for such a quench) satisfies $\varepsilon_{\text{res}} \sim t_Q^{-1/4}$; this is in perfect agreement with the result of Pellegrini *et al.* [31] for a similar linear quench of Δ . For a sudden quench of small magnitude (δ) from the gapless phase to the DCP, we find $\varepsilon_{\text{res}} \sim \delta^{3/2}$ (Inset Fig. 2). We emphasize that this is in perfect agreement with the adiabatic perturbation theoretic prediction [34] $\varepsilon_{\text{res}} \sim \delta^{\nu(d+z)}$ with $\nu = 1/2$ and $z = 2$; this implies that dynamical exponent associated with the DCP dictate the scaling of ε_{res} .

We next move to the spatial quench with Δ fixed to -1 and estimate ν_H . In the homogeneous system with $h_i = h, \forall i$, the ground state with $h = 0$ as well as $h \neq 0$ is the fully polarized state and the energy density of the ground state scales as $\varepsilon(h) - \varepsilon(0) = -|h|$. The energy gap, on the other hand, scales with h as $\Delta E \sim |h| \sim |h|^{\nu_H z}$ [35]; with $z = 2$, this immediately implies $\nu_H = 1/2$. We emphasize here that this value of ν_H differs from $1/(d+z)$ predicted before. This is due to the fact that the ground state is completely spin polarized without fluctuations resulting in a ξ_H that is independent of h and always ~ 1 . Consequently, the the ground-state energy den-

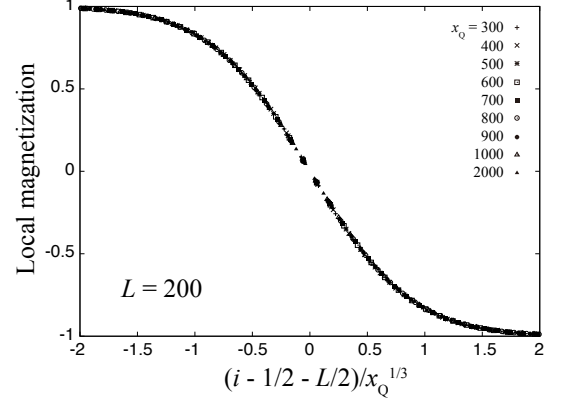


FIG. 3: Local magnetization obtained by the static DMRG of the XXZ chain with $\Delta = -1$ and with a sloped magnetic field $h = (i - 1/2 - L/2)/x_Q$. The abscissa indicates the position scaled by $x_Q^{1/3}$. Results for several x_Q collapse into a single curve thereby confirming the predicted scaling.

sity does no longer scale as $\sim |h|^{(d+z)\nu_H}$, rather scales as $|h|^{z\nu_H}$; using this scaling one arrives at $\nu_H = 1/z$ and $\xi_s \sim x_Q^{1/(z+1)} = x_Q^{1/3}$ with $z = 2$.

Let us now apply a sloped field $h_i = (i - 1/2 - L/2)/x_Q$ choosing an even L and probe the local magnetization, $m_i = \langle \sigma_i^z \rangle$ of the ground state. The scaling theory presented above predicts that the characteristic length scale near $i = (L + 1)/2$ is given by $\xi_s \sim x_Q^{1/3}$. In fact, one then expects the scaling $m_i \sim \mathcal{F}\left(\frac{i - \frac{1}{2} - \frac{L}{2}}{x_Q^{1/3}}\right)$, where \mathcal{F} is the scaling function; remarkably this prediction perfectly matches with the numerical results obtained using static DMRG as presented in Fig. 3.

In conclusion, we provided universal scaling relations for quenches in the vicinity of a quantum DCP for both spatial and temporal quenches. Furthermore, we have established that indeed there exists a DCP in the phase diagram of a spin-1/2 XXZ chain. The critical exponents and hence the scaling relations following quenches get non-trivially modified since this DCP occurs at the terminating point of a gapless critical line. Most importantly, we posit generic scaling relations which perfectly reduce to both the situations (Ising and XXZ) considered here. We emphasize that the scaling predictions for the XXZ chain are verified numerically. An experimental demonstration of our results is expected in a cold atomic system in an optical lattice or a trapped-ion system.

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Supplemental Material on “Universal scaling in quenches across a discontinuity critical point”

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Here, we shall establish that the quantum critical point appearing at the boundary of the gapless critical line and the gapped ferromagnetic phase of the spin-1/2 XXZ chain (Hamiltonian (9) of the main text) with $h_i = 0, \forall i$,

$$H = \sum_{i=1}^{L-1} (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^x \sigma_{i+1}^z) \quad (S1)$$

at $\Delta = \Delta_c = -1$ is indeed a discontinuity critical point (DCP) [S1, 2] and show how the associated critical exponents get altered in this continuous symmetry model where the DCP is located at the boundary of a gapless critical line; in the process, we shall also touch upon the scaling relations and critical exponents associated with a generic DCP. The complete phase diagram of the chain is shown in Fig. S1 [S3].

To this end, we recall that the ground-state energy density $\varepsilon(\Delta)$ in the $\sigma_{\text{tot}}^z = 0$ manifold is given exactly by [S4]

$$\varepsilon(\Delta) = \begin{cases} \Delta & (\text{for } \Delta < -1) \\ \cos \mu - \sin^2 \mu \int_{-\infty}^{\infty} \frac{dx}{\cosh \pi x (\cosh 2\mu x - \cos \mu)} & (\text{for } -1 < \Delta < 1) \end{cases}, \quad (S2)$$

where μ is defined through the relation $\cos \mu = \Delta$. When $0 < \Delta + 1 \ll 1$, making an expansion $\cos \mu \approx -1 + \frac{1}{2}\delta^2$

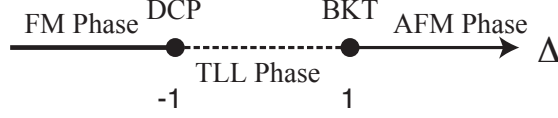


FIG. S1: The phase diagram of Hamiltonian (S1); the gapless region lying between $-1 < \Delta \leq 1$ is the Tomonaga-Luttinger liquid (TLL) phase. The transition between the gapless phase and the antiferromagnetically (AFM) ordered phase at $\Delta = 1$ is Berezinski-Kosterlitz-Thouless (BKT) transition point while the critical point separating the gapless phase from the ferromagnetic (FM) phase at $\Delta = -1$ is a discontinuity critical point (DCP).

(which implies $\frac{1}{2}\delta^2 \approx \Delta + 1$) and retaining lowest order terms in Taylor expansion in δ , we arrive at

$$\begin{aligned}\varepsilon(\Delta) &\approx -1 - \frac{1}{3\pi}\delta^3 + \mathcal{O}(\delta^4) \\ &= -1 + \frac{2\sqrt{2}}{3\pi}(\Delta + 1)^{3/2} + \mathcal{O}((\Delta + 1)^2).\end{aligned}$$

Consequently, one gets

$$\varepsilon(\Delta) - \varepsilon(\Delta_c) \approx \begin{cases} \Delta - \Delta_c & (\text{for } \Delta < -1) \\ \frac{2\sqrt{2}}{3\pi}(\Delta - \Delta_c)^{3/2} & (\text{for } -1 < \Delta < 1) \end{cases}, \quad (\text{S3})$$

where we have used $\Delta_c = -1$. This immediately reveals that the derivative of $\varepsilon(\Delta) - \varepsilon(\Delta_c)$ is discontinuous at $\Delta = \Delta_c$, and establishes that this point is a DCP.

Now, in order to settle the value of ν , we compare $\varepsilon(\Delta) - \varepsilon(\Delta_c) \sim (\Delta - \Delta_c)^{3/2}$ for $\Delta > \Delta_c$ with the conventional scaling of the ground state energy density[S5, 6], $(\Delta - \Delta_c)^{(d+z)\nu}$. This comparison leads us to the relation $(d+z)\nu = 3/2$. Note here $z = 2$, since the spin wave accounts for the excitation at $\Delta = \Delta_c$ even in the $\sigma_{\text{tot}}^z = 0$ manifold and the spectrum is quadratic in the momentum. Therefore, taking $d = 1$ into account, one finds $\nu = 1/2$. Surprisingly, this is inconsistent with the prediction $1/(d+z) = 1/3$. As we shall elaborate below, this anomaly emerges from the existence of a critical line for $\Delta > \Delta_c$.

To modify the scaling theory accordingly, we assume that the spectrum follows the form $|\Delta - \Delta_c|^{\theta_{\pm}} k^{z_{\pm}} + k^z$, where (θ_{-}, z_{-}) and (θ_{+}, z_{+}) represent exponents on the lower- Δ and the higher- Δ sides of the DCP, respectively. We here demand $0 \leq z_{\pm} < z$. Clearly, one can readily retrieve the quadratic scaling of the gap right at the DCP by setting $\Delta = \Delta_c$. Comparison of the spectrum with the conventional scaling form $|\Delta - \Delta_c|^{\nu z}$ yields

$$\theta_{\pm} = \nu(z - z_{\pm}). \quad (\text{S4})$$

Next, we assume that the correlation length has the form $\xi(\Delta) \sim 1 + A_{\pm}|\Delta - \Delta_c|^{-\nu}$, where \pm corresponds to the sign of $\Delta - \Delta_c$. It is to be noted that $\xi(\Delta) \sim 1$ when $A_{-} = 0$ (or $A_{+} = 0$); this situation arises when the fluctuation of the order parameter is entirely absent which indeed happens, as we shall show below, in the ground state of the XXZ chain for $\Delta < \Delta_c$. Having assumed the the scaling form of the spectrum and the correlation length, the ground-state energy density can be written as

$$\varepsilon(\Delta) - \varepsilon(\Delta_c) \sim |\Delta - \Delta_c|^{\theta_{\pm} + \nu z_{\pm}} \xi(\Delta)^{-d} \sim |\Delta - \Delta_c|^{\theta_{\pm} + \nu z_{\pm}} (1 + A_{\pm}|\Delta - \Delta_c|^{-\nu})^{-d}. \quad (\text{S5})$$

We emphasize that, in general, this quantity does not scale with an identical exponent on the lower- and higher- Δ sides. To resolve this asymmetry, let us introduce the new quantity:

$$\varphi(\Delta) = \begin{cases} (\varepsilon(\Delta) - \varepsilon(\Delta_c)|\Delta - \Delta_c|^{\nu z_{-}}) & \text{for } \Delta < \Delta_c \\ (\varepsilon(\Delta) - \varepsilon(\Delta_c)|\Delta - \Delta_c|^{\nu z_{+}}) & \text{for } \Delta > \Delta_c \end{cases}, \quad (\text{S6})$$

and demand modified conditions of the DCP as follow: (i) this quantity follows the same scaling relation on both

sides of the critical point and that (ii) it is continuous while (iii) its derivative is discontinuous at the critical point:

$$(i) \quad \varphi(\Delta_c - \epsilon) \sim \varphi(\Delta_c + \epsilon) \sim |\epsilon|^a \quad \text{with an } a > 0, \quad (S7)$$

$$(ii) \quad \varphi_-(\Delta_c - 0) = \varphi_+(\Delta_c + 0), \quad (S8)$$

$$(iii) \quad \frac{\partial \varphi_-(\Delta_c - 0)}{\partial \Delta} \neq \frac{\partial \varphi_+(\Delta_c + 0)}{\partial \Delta}. \quad (S9)$$

We hereafter discuss the cases with $A_- = 0$ and $A_- \neq 0$ separately, with $A_+ \neq 0$ in both the situations.

We first consider the situation where $A_- = 0$. Equations (S5) and (S6) then yield

$$\varphi(\Delta) \sim \begin{cases} (\Delta_c - \Delta)^{\theta_-} & \text{for } \Delta < \Delta_c \\ (\Delta - \Delta_c)^{\theta_+ + d\nu} & \text{for } \Delta > \Delta_c \end{cases}, \quad (S10)$$

Using Eq. (S4) and the condition (i) on $\varphi(\Delta)$, one obtains $(z - z_-)\nu = (d + z - z_+)\nu$, which immediately reduces to $z_- = z_+ - d$. The condition (ii) along with (i) result in constraint relations, $z_- \leq z$ and $z_+ \leq d + z$ (though these are always satisfied under the restriction we have imposed on z_{\pm}). Finally, by the condition (iii), one obtains $\nu = 1/(d + z - z_+) = 1/(z - z_-)$. It is noteworthy that we are now left with only two independent exponents out of the six exponents z , ν , z_{\pm} , and θ_{\pm} involved in the scaling. For instance, if z and z_+ are known for a given d , the other exponents are automatically fixed. Now let us return to the XXZ chain. The ground-state energy density and the energy gap of the chain scale identically as $|\Delta - \Delta_c|$ for $\Delta < \Delta_c$. This scaling together with the fact that the spectrum is gapped in this region imply $A_- = 0$ and establish that the XXZ chain indeed corresponds to the present situation. Applying the value $d = z_+ = 1$ and $z = 2$ of the XXZ chain to the generic relations, one obtains $\nu = 1/2$, $z_- = 0$, $\theta_- = 1$, and $\theta_+ = 1/2$, which are completely consistent with the property of the XXZ chain.

Next, we consider the case with $A_- \neq 0$. Equations (S5) and (S6) now reduce to

$$\varphi(\Delta) \sim \begin{cases} (\Delta_c - \Delta)^{\theta_- + d\nu} & \text{for } \Delta < \Delta_c \\ (\Delta - \Delta_c)^{\theta_+ + d\nu} & \text{for } \Delta > \Delta_c \end{cases}. \quad (S11)$$

Following the same set of arguments as in the previous situation results in $z_- = z_+$ and $\nu = 1/(d + z - z_+) = 1/(d + z - z_-)$. If one sets $z_+ = 0$, one obtains $\nu = 1/(d + z)$. Hence the correlation length exponent of a generic isolated DCP, mentioned in the main text, is retrieved.

To summarize, we have provided the scaling theory for a generic DCP based on the conditions mentioned above. When a DCP characterized by the dynamical exponent z (> 0) is located between two critical phases with different dynamical exponents z_- and z_+ (assuming $0 \leq z_- < z_+ < z$ without loss of generality), the exponents z_- and z_+ are constrained by $z_+ - z_- = d$ and the correlation length exponent is given by $\nu = 1/(d + z - z_+) = 1/(z - z_-)$. Moreover, the critical amplitude of the correlation length satisfies the condition $A_-/A_+ = 0$. Note that these rules are valid even when one of the two phases is gapped ($z_- = 0$). On the other hand, when a DCP with $z > 0$ is located between two critical phases with an identical exponent $z_- = z_+ = z' < z$ or two gapped phases ($z_- = z_+ = z' = 0$), one finds $\nu = 1/(d + z - z')$ and the critical amplitudes of the correlation length are finite on the both sides of the DCP.

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